Surface-based Morphometry using SPHARM

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Surface-based Morphometry

Thalamus shape in Multiple Sclerosis

Analyzed by SPHARM-MAT and SurfStat
SPHARM-MAT for Surface Modeling

- NIBIB R03 Project
  - Shape analysis toolkit for neuroimaging studies
- Major components
  - Spherical parameterization
  - SPHARM expansion
  - SPHARM alignment
- Interface with other tools
  - SPHARM-PDM
  - SurfStat
  - 3D Slicer

SPHARM Modeling

\[ \mathbf{v}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{lm}^{m} Y_{lm}^{m}(\theta, \phi) \]

\[ \mathbf{v}(\theta, \phi) = \begin{pmatrix} x(\theta, \phi) \\ y(\theta, \phi) \\ z(\theta, \phi) \end{pmatrix} \quad \mathbf{c}_{lm}^{m} = \begin{pmatrix} c_{lm}^{m} \\ \vdots \end{pmatrix} \]

Area-preserving mapping

(\theta, \phi) \leftrightarrow (x, y, z)
(1) Spherical Parameterization

\[(x_2 - x_1) \times (y - y_1) - (x - x_1) \times (y_2 - y_1) = 2A_1\]

\[(x_3 - x_2) \times (y - y_2) - (x - x_2) \times (y_3 - y_2) = 2A_2\]

\[(x_4 - x_3) \times (y - y_3) - (x - x_3) \times (y_4 - y_3) = 2A_3\]

\[(x_5 - x_4) \times (y - y_4) - (x - x_4) \times (y_5 - y_4) = 2A_4\]

\[(x_1 - x_5) \times (y - y_5) - (x - x_5) \times (y_1 - y_5) = 2A_5\]

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CALD Spherical Parameterization

**Algorithm**

CALD spherical parameterization.

1. perform initial parameterization
2. perform \(n\) steps of local smoothing
3. **repeat**
4. perform one step of global smoothing
5. perform \(n\) steps local smoothing
6. **until** stop criterion is achieved
(2) SPHARM Expansion

- **SPHARM expansion:**
  \[
  \mathbf{v}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_l^m Y_l^m(\theta, \phi),
  \text{ where } c_l^m = \begin{pmatrix} c_{lx}^m, c_{ly}^m, c_{lz}^m \end{pmatrix}^T.
  \]

- **Naïve Least Square Fitting (NLSF):**
  \[
  \hat{f}(\theta, \phi) = \sum_{l=0}^{L_{\text{max}}} \sum_{m=-l}^{l} \hat{a}_l^m Y_l^m(\theta, \phi) \approx f(\theta, \phi).
  \]

Not suitable for large values of \(L_{\text{max}}\) (max degree) and \(n\) (# of surface samples), since the problem is to solve an \(n \times (L_{\text{max}}+1)^2\) linear system.

**Iterative Residual Fitting (IRF)**

- Since SHs form a coarse-to-fine hierarchy, we first extract low frequency components and then use the residual to get high frequency components.

1. Solve the linear system:
   \[
   (A_0 A_1 \ldots A_{L_{\text{max}}}) (b_0^T \ b_1^T \ldots b_{L_{\text{max}}}^T)^T = f.
   \]
2. Calculate the residual:
   \[
   r = f - (A_0 A_1 \ldots A_{L_{\text{max}}}) (b_0^T \ b_1^T \ldots b_{L_{\text{max}}}^T)^T.
   \]
3. Iteratively fit the residual:
   \[
   \text{for } l = s + 1; \ l \leq L_{\text{max}}; \ l \rightarrow l + 1 \text{ do}
   \]
   solve for \(A_l b_l = r\)
   update residual \(r = r - A_l b_l\)
4. Return the spherical harmonic model \(m:\)
   \[
   m = (b_0^T \ b_1^T \ldots b_{L_{\text{max}}}^T)^T.
   \]
(3) SPHARM Registration

(a) Original objects (b) aligned parameterizations (c) aligned objects

Works only if FOE is a real ellipsoid

SHREC

- Naive solution
  - Recalculate the SPHARM coefficients using the rotated parameterization.
  - Solving three linear systems is time-consuming

- Fast method
  - Use a rotational property
  - Rotate SPHARM coefficients directly
  - Without recalculating the SPHARM expansion.
Landmark Guided Registration

- Preserve homological properties
- Employ spherical thin plate spline
- Optimize landmark placement on sphere

(4) Statistical Shape Analysis

- Thalamic shape in MS
  - MS (n=25) vs HC (n=12)
- Atlas generation and surface signal extraction
Linear Model using SurfStat

- SPHARM modeling
  - Spherical parameterization: CALD
  - SPHARM expansion: IRF
  - SPHARM registration: SHREC, STPS
- Statistical shape analysis
  - Interface with SurfStat

Summary
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