Surface-based Morphometry using SPHARM

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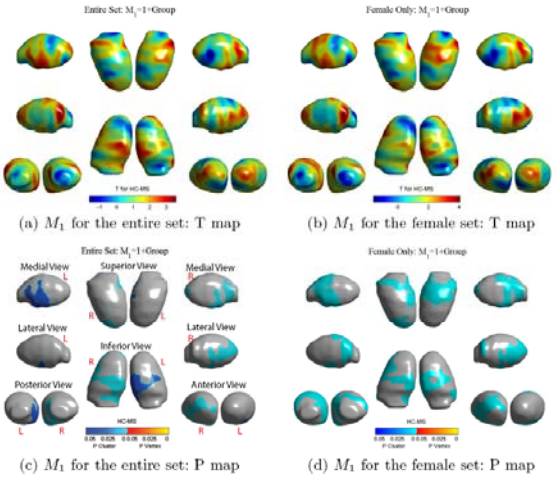
Surface-based Morphometry

Thalamus shape in Multiple Sclerosis

Analyzed by SPHARM-MAT and SurfStat

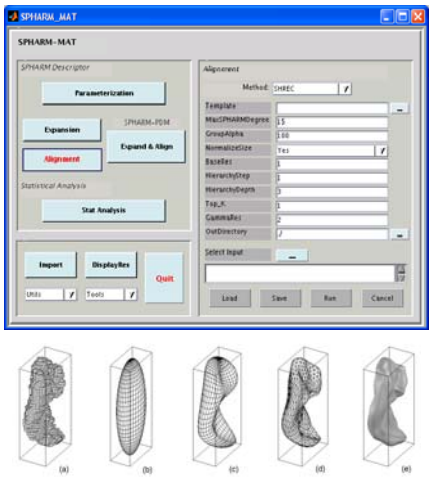


(a) M_1 for the entire set: T map (b) M_1 for the female set: T map

(c) M_1 for the entire set: P map (d) M_1 for the female set: P map

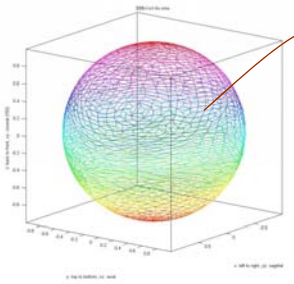
SPHARM-MAT for Surface Modeling

- NIBIB R03 Project
  - Shape analysis toolkit for neuroimaging studies
- Major components
  - Spherical parameterization
  - SPHARM expansion
  - SPHARM alignment
- Interface with other tools
  - SPHARM-PDM
  - SurfStat
  - 3D Slicer



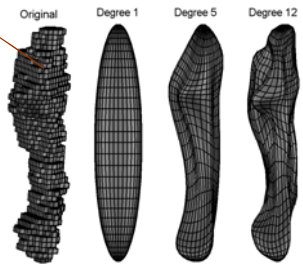
SPHARM Modeling

$$\mathbf{v}(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \mathbf{c}_l^m Y_l^m(\theta,\phi) \qquad \mathbf{v}(\theta,\phi) = \begin{pmatrix} x(\theta,\phi) \\ y(\theta,\phi) \\ z(\theta,\phi) \end{pmatrix} \qquad \mathbf{c}_l^m = \begin{pmatrix} c_{xl}^m \\ c_{yl}^m \\ c_{zl}^m \end{pmatrix}$$

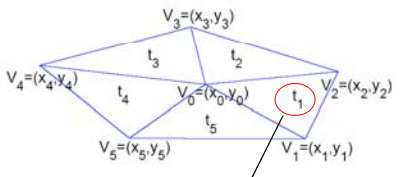


$(\theta,\phi) \leftrightarrow (x,y,z)$

Area-preserving mapping



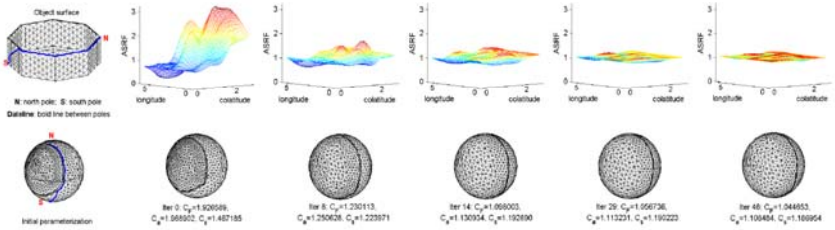
(1) Spherical Parameterization



$$
\begin{aligned}
(x_2 - x_1) \times (y - y_1) - (x - x_1) \times (y_2 - y_1) &= 2A_1 \\
(x_3 - x_2) \times (y - y_2) - (x - x_2) \times (y_3 - y_2) &= 2A_2 \\
(x_4 - x_3) \times (y - y_3) - (x - x_3) \times (y_4 - y_3) &= 2A_3 \\
(x_5 - x_4) \times (y - y_4) - (x - x_4) \times (y_5 - y_4) &= 2A_4 \\
(x_1 - x_5) \times (y - y_5) - (x - x_5) \times (y_1 - y_5) &= 2A_5
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{1}{2} \times \begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \\
&= \frac{1}{2}((x_2 - x_1) \times (y_0 - y_1) - (x_0 - x_1) \times (y_2 - y_1))
\end{aligned}
$$

CALD Spherical Parameterization



Algorithm CALD spherical parameterization.

1: perform initial parameterization
2: perform n steps of local smoothing
3: **repeat**
4: perform one step of global smoothing
5: perform n steps local smoothing
6: **until** stop criterion is achieved

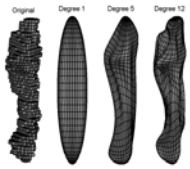
(2) SPHARM Expansion

- **SPHARM expansion:**

$$\mathbf{v}(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \mathbf{c}_l^m Y_l^m(\theta,\phi), \quad \text{where } \mathbf{c}_l^m = \left(c_{lx}^m, c_{ly}^m, c_{lz}^m\right)^T.$$

- **Naïve Least Square Fitting (NLSF):**

$$\hat{f}(\theta,\phi) = \sum_{l=0}^{L_{max}} \sum_{m=-l}^{l} \hat{a}_l^m Y_l^m(\theta,\phi) \approx f(\theta,\phi).$$

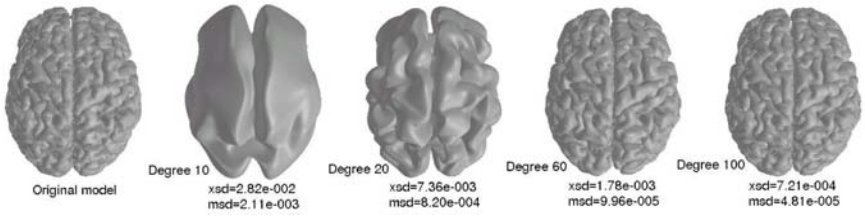


Not suitable for large values of L_{max} (max degree) and n (# of surface samples), since the problem is to *solve an n × $(L_{max}+1)^2$ linear system*.

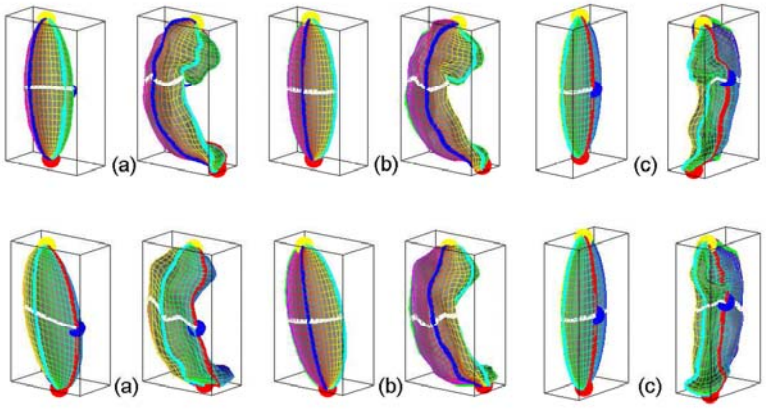
Iterative Residual Fitting (IRF)

- Since SHs form a coarse-to-fine hierarchy, we first extract low frequency components and then use the residual to get high frequency components.

1. Solve the linear system:
   $(A_0\ A_1\ \ldots\ A_s)\ (\mathbf{b}_0^T\ \mathbf{b}_1^T\ \ldots\ \mathbf{b}_s^T)^T = \mathbf{f}$.
2. Calculate the residual:
   $\mathbf{r} = \mathbf{f} - (A_0\ A_1\ \ldots\ A_s)\ (\mathbf{b}_0^T\ \mathbf{b}_1^T\ \ldots\ \mathbf{b}_s^T)^T$.
3. Iteratively fit the residual:
   for $(l = s+1; l \le L_{max}; l++)$ **do**
   solve for $A_l\mathbf{b}_l = \mathbf{r}$
   update residual $\mathbf{r} = \mathbf{r} - A_l\mathbf{b}_l$
4. Return the spherical harmonic model $\mathbf{m}$:
   $\mathbf{m} \equiv (\mathbf{b}_0^T\ \mathbf{b}_1^T\ \ldots\ \mathbf{b}_{L_{max}}^T)^T$.

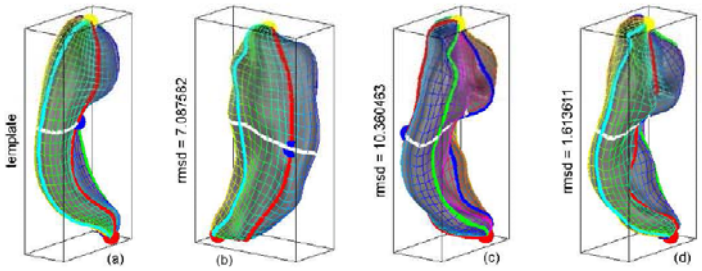


(3) SPHARM Registration



(a) Original objects (b) aligned parameterizations (c) aligned objects
Works only if FOE is a real ellipsoid

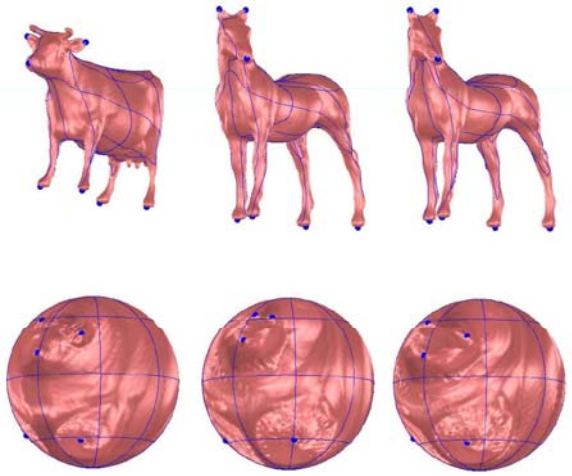
SHREC



- Naive solution
  - Recalculate the SPHARM coefficients using the rotated parameterization.
  - Solving three linear systems is time-consuming
- Fast method
  - Use a rotational property
  - Rotate SPHARM coefficients directly
  - Without recalculating the SPHARM expansion.

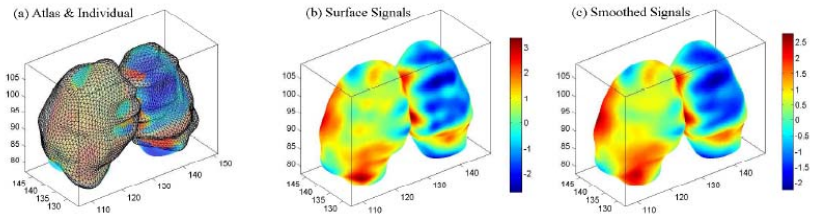
Landmark Guided Registration

- Preserve homological properties
- Employ spherical thin plate spline
- Optimize landmark placement on sphere

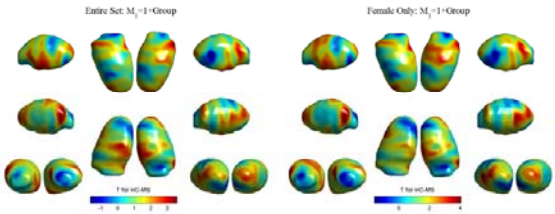


(4) Statistical Shape Analysis

- Thalamic shape in MS
  - MS (n=25) vs HC (n=12)
- Atlas generation and surface signal extraction

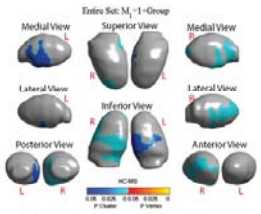


Linear Model using SurfStat

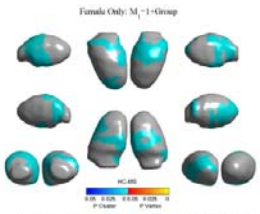


(a) M_1 for the entire set: T map

(b) M_1 for the female set: T map



(c) M_1 for the entire set: P map



(d) M_1 for the female set: P map

Summary

- SPHARM modeling
  - Spherical parameterization: CALD
  - SPHARM expansion: IRF
  - SPHARM registration: SHREC, STPS
- Statistical shape analysis
  - Interface with SurfStat

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